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Curriculum I.12D
Four-Year Program

ONTARIO DEPARTMENT OF EDUCATION

MATHEMATICS

INTERMEDIATE DIVISION

Grade 10

Four-Year Program

All Branches

30M-66—G.12932

MATHEMATICS

INTRODUCTION

1. This course is designed for students of the Four-Year Program in all Branches who have completed the earlier courses of this sequence, that is Curriculum I.12A (Grade 7), Curriculum I.12B (Grade 8), and Curriculum I.12C (Grade 9, Four- or Five-Year Program). Use for other pupils must take account of the sections of these courses not previously studied.

2. This course provides students of the Four-Year Program with fundamental mathematics on which further mathematics education can be based, and also provides specific topics of interest and use to pupils in the various Branches. The course consists of two parts: a core program to be studied by all students, and specific optional topics — at least two of which together with the core constitute the year's work. The choice of optional topics is to be made according to the branch in which the students are registered; the Business and Commerce Branch will choose from Units 6, 7 and 8. It is not necessary that the order of the topics in this outline be followed in presenting the course to pupils. It is recommended that the course begin with Unit 1, Computation, so that the principles and procedures developed in it will be of use to the student throughout subsequent parts of the course, and that where possible a spiral approach be used within the course to integrate topics from different units.

3. All pupils are expected to cover the topics outlined in:

   UNIT 1: Computation
   UNIT 2: Irrational Numbers and Square Roots
   UNIT 3: Relations and Functions
   UNIT 4: Purchasing Personal Property
   UNIT 5: Ratio and Proportion

together with a least two of:

UNIT 6: Managing Personal Property
UNIT 7: Payroll
UNIT 8: Systems of Numeration
UNIT 9: Operations with Polynomials
UNIT 10: Numerical Applications of Geometry
UNIT 11: Trigonometry
UNIT 12: Geometric Vectors

Where possible, the opportunity should be provided for better students to pursue more than two of the optional units.

4. The "Teaching Suggestions" on pages 7 to 25 are included as an assistance to teachers. These notes are ***not*** to be considered to provide the only interpretation of the course nor to deal with all the topics. ***They should in no way inhibit originality of approach by the teacher nor the adaptation of this course to local needs.***
5. The suggested timing in the course is provided as an indication of the depth to which the various topics should be pursued. It is based upon five periods per week.
6. Throughout the course, the examples and problems should be practical and should be directed to the pupils' interests and experiences.
7. It is expected that four-figure mathematical tables with values of the trigonometric functions for 1° intervals will be used.

UNIT 1:
COMPUTATION (5 weeks)

This unit extends the work on exponents and introduces useful computational procedures and mechanical aids while providing an opportunity to review the work of previous grades on mensuration. A review of the material on the presentation and interpretation of data as outlined in Grade 8 Mathematics Unit 7 of Curriculum I.12B should be made, with extensions to show its use in polls and quality control in industry.

1. Rounding off, estimating, approximations

2. Slide Rule and 10-Key Calculator (mechanical use only)
3. Percent: definition, fractional and decimal equivalents; problems involving use of percent
4. Review of previous work on exponents
5. Negative integral exponents
6. Standard notation
7. Problems involving presentation and intepretation of data:
   (i) Presentation in tables
   (ii) Determination of frequency distribution
   (iii) Calculation of mean, median, and standard deviation
   (iv) Illustrative graphs

UNIT 2:

IRRATIONAL NUMBERS AND SQUARE ROOTS (3 weeks)

1. Review of the operations with rational numbers and of the properties of the rational number system
2. Rational numbers as periodic decimals
3. Irrational numbers as non-periodic decimals; their existence on the number line demonstrated by geometric construction
4. The set of real numbers: union of the set of rationals and the set of irrationals
5. Rational number approximations of irrational numbers and operations with these
6. Square roots
   (i) Definition of principal square root
   (ii) Operations with principal square roots:
   Multiplication, addition, subtraction, and conversions such as $\sqrt{8} = 2\sqrt{2}$, $\sqrt{2}$, $2\sqrt{3} = \sqrt{12}$
   Use of these relationships to find approximations of square roots such as $\sqrt{8}$, $\sqrt{18}$, and $\sqrt{27}$, given the approximations for $\sqrt{2}$ and $\sqrt{3}$

(iii) Square roots using slide rule
(iv) Use of square-root tables
(v) Problems requiring formulas involving square roots and simple powers

UNIT 3:
RELATIONS AND FUNCTIONS (5 weeks)

1. RELATIONS
   Introduction to binary relations (mathematical and other examples)
   Sets of ordered pairs; domain and range of a relation
   Cartesian product of two sets of real numbers, $R \times R$
   Graphing ordered pairs
   Relations in $R \times R$ defined by open sentences involving equations and inequations

2. FUNCTIONS
   Graphical approach; vertical line test, domain and range of a function
   The $f(x)$ notation and its use
   Functions defined by open sentences involving equations of the first degree in two variables, to discover that if $x \epsilon R$ the graphs of such equations are straight lines

3. Graphs of equations of the form $y = mx + b$ for numerical values of m and b, to discover the roles of m and b

4. Graphs of inequations of the forms $y > mx + b$, $y < mx + b$, $y \geqq mx + b$, $y \leqq mx + b$

5. Graphical solution of a system of two first-degree (linear) equations in two variables

6. Algebraic solution of a system of two first-degree equations in two variables by the method of comparison of ordinates

7. Problem solving (by the method of 6).

UNIT 4:
PURCHASING PERSONAL PROPERTY (3 weeks)

1. METHODS OF PURCHASING
   (a) Cash purchases
   (b) Credit buying
   (c) Instalment buying, contract
2. SOURCES OF FUNDS
   Bank, Credit Union, Finance Company
3. COMPARISON OF EFFECTIVE INTEREST RATES
   The use of the formula $r = \frac{2NI}{P(n+1)} \times 100$

UNIT 5:
RATIO AND PROPORTION (3 weeks)

1. Two- and three-term ratios
2. Equal ratios, leading to a definition of proportion
3. Direct variation as a linear function. Introduction of the variation symbol '$\propto$', graphs
4. Inverse variation, graphs
5. Problems from science, mensuration, and business involving ratio, proportion, and direct, inverse, and partial variation

UNIT 6:
MANAGING PERSONAL PROPERTY (3 weeks)

1. DETERMINATION OF INTEREST
   Savings accounts —
   banks, trust companies, credit unions
   Canada Savings Bonds — average yield
2. INSURANCE
   Life — calculation of premium on various kinds of policies, loan values, cash surrender values, optional values (lapsed policy, paid-up policy)
   Property — fire, extended coverage, theft, legal liability, combinations of these, premium, short-term rate table
   Automobile — calculating the premium from sample rate tables
3. COST OF AUTOMOBILE OWNERSHIP
   Operating expenses including repairs, insurance, depreciation, interest on investment; cost of operation per period and per mile

UNIT 7:

PAYROLL (3 weeks)

1. Determination of wages: hourly, piecework, bonus, incentive rates; overtime; deductions of various kinds
2. Preparation of payroll summary, earnings record cards, currency analysis; distribution of labour

UNIT 8:

SYSTEMS OF NUMERATION (3 weeks)

1. Numeration systems with bases other than 10: binary, octal, and hexadecimal
2. Conversion of numerals: decimal to binary, binary to decimal, binary to octal, octal to binary, decimal to octal, octal to decimal
3. Binary Coded Decimal System; alphabetic characters represented by BCD
4. Applications of above numeration systems

UNIT 9:

OPERATIONS WITH POLYNOMIALS (3 weeks)

1. Addition, subtraction, multiplication, and division with polynomials
2. Special products and factoring of Grade 9 extended to include the square of a binomial of the form $ax + b$, where a, b are rational, and factoring of a trinomial square
3. Solution of first-degree equations in one variable arising from products of polynomials

UNIT 10:

NUMERICAL APPLICATIONS OF GEOMETRY (3 weeks)

1. Review of congruency, parallelism, angle sum, and Pythagorean relationships
2. Numerical problems based on these relationships
3. Construction of solids; calculations necessary for construction of models of the regular polyhedra, cones, pyramids, and prisms

UNIT 11:

TRIGONOMETRY (3 weeks)

It is recommended that this unit be taught in the spring so that pupils can make measurements outside to produce data for problems.

1. Definition of the primary trigonometric ratios: $\sin \theta$ $\cos \theta$, and $\tan \theta$, where θ is any angle
2. Relationship of these ratios to the right-angled triangle
3. The use of tables of the values of the trigonometric ratios of acute angles (values for one-degree intervals)
4. Right triangle problems with the data collected from actual situations; use of transit or gravity protractor

UNIT 12:

GEOMETRIC VECTORS (3 weeks)

1. Review of basic constructions using ruler, compasses, and protractor
2. Vectors as directed line segments
3. Addition of vectors by scale drawings
4. Multiplication of a vector by a scalar
5. Extension to scale drawings for navigation and physics problems

SUGGESTIONS FOR TEACHERS

The following notes are included as an indication of the intent of the course of study and to provide some suggestions for its presentation. However, these notes should in no way inhibit originality of presentation nor adaptation of the course to local situations.

Students in the Four-Year Program respond best to varied and imaginative classroom techniques that are characterized by examples and illustrations of a practical nature. Formal developments and rigorous proofs may have to be minimized. However, understanding and the applications of basic mathematical principles are major objectives. These may be achieved by the study of suggestive patterns and thought-provoking examples which, wherever possible, should be related to the branch in which the students are registered.

UNIT 1:

COMPUTATION

Topic 1. Rounding off, Estimating, Approximations

This topic is primarily intended to give the student sufficient background for an understanding of many of the topics in the rest of the course. From it the student should learn that a measurement cannot be represented exactly by a numeral, and that approximate results are useful in many circumstances, and learn the use of rounding off in estimating and representing results.

Most of the suggestions that follow might well be introduced progressively during the course, by relating them to particular problems in other topics.

The student should become aware that real numbers are used to represent both exact and approximate data; also of situations in which both types of data occur. For example, the real number used to indicate the attendance at a particular concert may state exactly how many were present, or it may have been rounded off to express approximately how many. Numbers are often rounded off in order that they may not suggest a false sense of accuracy; for example, when giving the population of a city as 650,000.

The student should realize that when a number is read from a measuring instrument, such as a ruler, weighing scales or calipers, this number is an approximation of the measure in question. The more precise the instrument, the closer the approximation. A weight recorded as 25.34 grams indicates a measurement made to the nearest hundredth gram and should be interpreted as representing a weight between 25.335 grams and 25.345 grams.

It is important that pupils learn to make estimates of the results of calculations, so that they can readily check the reasonableness of their calculated results. For example, $\frac{42\times 88}{49}$ can be estimated as 70 by first rounding off to obtain $\frac{40\times 90}{50}$. The 'rounded off' numbers 40, 90, 50 and the estimate 70 are each approximations.

In problems involving measurements it should be understood that the measurements are as precise as stated,

and the pupil should be required to find to some stated precision the results of the calculations with these numbers; for example, to the nearest tenth, or hundredth.

The method used to round off decimal numerals should be illustrated, and applied in examples throughout the course.

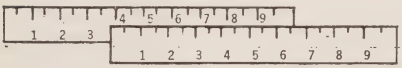
Approximations arise in many places in the course; for example, in most practical situations, and in rational approximations for irrational numbers such as π, $\sqrt{2}$, and sin 60°.

Exercises should include ones requiring reasons for rounding off numbers.

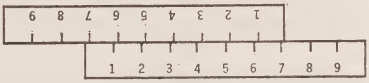
This topic could be presented by a lab-workshop approach.

TOPIC 2. SLIDE RULE AND 10-KEY CALCULATOR
(mechanical use only)

The notion of mechanical calculators might be introduced by using two rulers (preferably scaled in tenths) to perform addition or subtraction by the methods illustrated below.



Addition $380 + 460 = 840$



Subtraction $710 - 420 = 290$

This would require that the student learn to read the ruler scale as 0,1,2,3, ···, 10 or 0,10,20,30, ···, 100 or 0,100,200, ···, 1,000, etc. and so this would prepare him for interpretations of the scales on the (logarithm) slide rule, i.e., the

A, B scales from 1 to 100 or 100 to 10,000, etc.
C, D scales from 1 to 10 or 10 to 100, etc.
(Supplementary — K scale from 1 to 1,000 or 1,000 to 1,000,000, etc.)

The use of a transparent slide rule with an overhead projector and/or a large demonstration slide rule would facilitate class instruction.

The slide rule may now be used to find: squares (and cubes), the removal of the slide facilitates this operation; products; quotients (using C, D and/or CI, D scales).

This first introduction to the slide rule should *not* be exhaustive. At this time the examples should be simple and build the student's confidence. There are numerous other opportunities in the course in which to extend its applications and to introduce more difficult examples.

The student should become familiar with the approximate nature of results obtained from a slide rule, and at this stage he should position the decimal point by estimation of the result. Exercises such as $\frac{.00493 \times .000872}{.0325}$ should not be considered until after Topic 6 has been covered.

Exercises should include problems using familiar mensuration formulas, and should also introduce new formulas such as $V = \frac{4}{5}\pi r^3$.

The operations of addition, subtraction, multiplication and division on a 10-key calculator should be demonstrated to the students, and then time allotted to individual students for practice sessions on the available machines. The students should be encouraged to use the machines throughout the year for computations in their assignments. Ideally, the student should be given assignments which would progressively familiarize him with the techniques of the calculator and at the same time help him to recall previously established concepts.

Topic 3. Percent: Definition, Fractional and Decimal Equivalents

Problems Involving Use of Percent

This topic reviews the work on percent of previous grades, while giving the student practice in simple problems with the slide rule.

Topics 4 and 5.

Review of Previous Work on Exponents

The meaning of x^n where n is a natural number and it is understood that x is a rational number should be reviewed from many numerical examples leading to the

definition of x^n. The following power laws should also be reviewed from examples in which the exponents are natural numbers, and then the laws themselves established intuitively as a general expression of the obvious pattern.

Laws for exponents: If $m, n \in N$, then

(i) $x^m.x^n = x^{m+n}$;

(ii) $(xy)^n = x^n.y^n$;

(iii) $\frac{x^m}{x^n} = x^{m-n}$ where $m > n, x \neq 0$;

(iv) $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ where $y \neq 0$;

(v) $(x^m)^n = x^{mn}$.

Definitions for x^0 and x^{-n} *where* $n \in N$, $x \neq 0$ can now be determined so that the above laws will still hold. For example, by definition $\frac{3^4}{3^4} = \frac{3 \cdot 3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 3 \cdot 3} = 1$, but if (iii) is to hold when the exponents are equal, then $\frac{3^4}{3^4} = 3^{4-4} = 3^0$ and the definition $3^0 = 1$ follows. Many similar examples should be demonstrated before generalizing that $x^0 = 1$, when $x \neq 0$. A similar pattern will develop the meaning of x^{-n}.

Topic 6. Standard Notation

This is a review topic, built this year on a stronger foundation and with greater emphasis on positive rational decimals less than 1, such as $.256 = 2.56 \times 10^{-1}$ and $.0034 = 3.4 \times 10^{-3}$. Distinction should be made between numbers such as 2×10^3 and 2.00×10^3. Practice should be given using the slide rule for multiplication and division with such numbers. The position of the decimal point can now be determined in the more involved calculations by using standard notation.

Topic 7. Problems involving presentation and interpretation of data

This topic is intended to review and extend the work begun in grade 8. In industry, inspectors make tests on samples and record the data; the results are compared with standard patterns in order to study deviations.

In (iii) of Topic 7, if M is the mean of $x_1, x_2, x_3, \cdots, x_n$ then $M = \frac{x_1 + x_2 + \cdots + x_n}{n}$ and the standard deviation is given by $\sqrt{\frac{(M-x_1)^2 + (M-x_2)^2 + \cdots + (M-x_n)^2}{n}}$. The mechanical calculator should be used for calculations using these formulas. Students should collect and prepare the data for some of the exercises in this topic.

UNIT 2.
IRRATIONAL NUMBERS AND SQUARE ROOTS

Topic 1. Review of the Set of Rational Numbers, Operations with Rational Numbers, and the Properties of the Rational Number System

This reviews the first three items of Unit 3 of Curriculum I.12C, Four-Year Program. This could be done by recalling the extension of the set of natural numbers to (i) the set of whole numbers, (ii) the set of integers, and (iii) the set of rational numbers, in order to find non-empty solution sets of equations such as (i) $x + 3 = 3$, (ii) $x + 3 = 0$, and (iii) $3x + 4 = 0$, respectively. From this emerges the need for the set of rational numbers, the definitions of multiplication, division, addition, subtraction, and the properties of the rational number system. Adequate time should be allowed for practice in manipulations.

Topic 2. Rational Numbers as Periodic Decimals

The term 'periodic decimal' follows readily from a review of terminating decimals and non-terminating, repeating decimals, first introduced in Grade 9. Care should be taken to show that the decimal representation of a rational number such as $\frac{4}{5}$ is periodic; i.e., $.8\dot{0}$. The student should practise converting rational numbers of the form $\frac{a}{b}$ into periodic decimal form; and it should be illustrated that the inverse operation holds (but it is not expected that he should master the techniques). It might be shown to the student that $.75\dot{0}$ and $.74\dot{9}$ represent the same rational number. Some rational numbers should be graphed on a number line, and the density of the set of rational numbers discussed.

Topic 4. The Set of Real Numbers: Union of the Set of Rationals and the Set of Irrationals

This follows readily from Topic 3, and the graphing of some rational and irrational numbers. The student is asked to accept

(i) the notion of "completeness"; i.e. the one-to-one correspondence of the elements in the set of real numbers and the points in a line,

(ii) that the (field) properties hold; i.e. that $\sqrt{2} + \sqrt{3}$ is a real number, $\pi\sqrt{2}$ is real, $\pi + 3\sqrt{2} = 3\sqrt{2} + \pi$, etc.

Topic 5. Rational Number Approximations of Irrational Numbers and Operations with These

In calculations (practical applications) it is often convenient to use numerals which represent rational approximations of irrational numbers. For example:

(i) The irrational number π is often approximated by $\frac{22}{7}$ or 3.14 (correct to the nearest hundredth), 3.142, 3.1416, 3.14159 (correct to the nearest thousandth, etc.)

(ii) The real number $(\sqrt{2} + \sqrt{3})$ may be represented in decimal notation as follows:

$$\begin{aligned}\sqrt{2} + \sqrt{3} &\doteq 1.414 + 1.732 \\ &= 3.146\end{aligned}$$

where the rational number 3.146 is an approximation of $\sqrt{2} + \sqrt{3}$.

Topic 6. Square Roots

Practice should be given in finding square roots using the slide rule, square-root tables, and Newton's method with the 10-key calculator. It should be emphasized that the entires in the square-root tables in most cases represent rational approximations. Practice should be given in making whole-number estimations of principal square roots. This will enable the student to position the decimal point in his results.

The exercises should include suitable problems that require formulas involving square roots and powers (not necessarily restricted to those previously studied). Some formulas applicable to these problems are $a^2 = b^2 + c^2$, $W = E^2/R$, $A = \pi r^2$, $A = s^2$, $V = s^3$, $V = s^2h$, $V = \pi r^2 h$, $H.P. = \frac{D^2N}{2.5}$, $V = \frac{4}{3}\pi r^3$, and $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{a+b+c}{2}$ and a, b, c are measures of the sides of the triangle.

UNIT 3:
RELATIONS AND FUNCTIONS

The notions of relation and function are basic to the study of mathematics. It is not intended in this course that the student acquire a formal grasp of these ideas, but it is important that he sense their meanings. The student should already be familiar with ordered pairs, sets of ordered pairs, and graphs of simple equations from Unit 9 of Curriculum I.12C, Four-Year Program. This unit extends these ideas to the next level of sophistication; i.e., relations and functions. In his daily experiences he associates faces with names, players with teams, students with classes, cars with manufacturers, etc. Associations such as these can serve as an introduction to the 'set of ordered pairs' notion of relation. This easily extends to mathematical examples.

Topic 1. Relations

The significances of *ordered* pair, domain and range of a relation, and graph of a relation should be established from both mathematical and non-mathematical examples. The student should discover that the mathematical notations merely summarize ideas that were present in his mind all the time.

The cartesian product of two sets (whether finite or infinite) can be related to the student's experiences. $M \times C$ might represent the cross product of the set, M, of three models of cars and the set C, of five exterior colours. This could be extended with better classes to include interior colours, etc. – i.e., to a cross product of the form

$M \times C \times I \times \cdots$. The student will readily recognize in the above example that $M \times C$ contains 15 elements, and class discussion may reveal that in reality the relation available may be a proper subset of $M \times C$ (not all models can be obtained in all five colours). Of course the above figures, 3 and 5, are unrealistic. As a class project, students could undertake to determine the true picture for each of the major manufacturers of cars, and/or other products, and to represent them in tabular and graphical form.

The notion of cartesian product can now be readily extended to mathematical examples involving sets of real numbers (first finite, then infinite), and the meanings of $N \times N$, $I \times I$, $R \times R$ should be established.

Finally, examples should be introduced involving relations in $R \times R$ determined by open sentences involving equations or inequations such as $y = 3x + 1$, $y > 3x$, $y = x^2$, $x^2 = y$, and $y = \frac{1}{x}$ A table of values should be obtained to display some of the ordered pairs of the relation, and subsequently the graph drawn.

Topic 2. Functions

The idea of function is not intended to be treated formally, but rather as a special type of relation characterized by the vertical line test. This could follow from examining the graphs of the relations of the previous exercise to discover those which pass the function test, and would lead to an understanding of function in terms of the vertical line test, and/or of ordered pairs. The student should realize that all functions are relations, but not vice versa.

The $f(x)$ notation for 'the value of the function at x should be introduced by numerical values for x, and then generalized. It should be noted that for each x, $f(x)$ is unique. The emphasis here should be on the calculation of values such as $f(0)$, $f(-3)$, etc. It is not intended that calculations of the form $f(g(x))$ be included.

By using tables of values and drawing the graphs of relations defined by equations such as $2x + 3y = 6$, $y = 5x$ where $x \in R$, ($x = c$, $c \in R$ excluded), the student should discover that such a relation is always a function and that its graph is a straight line. This should lead to the convenient two-point method of graphing a function defined by a first degree equation in two variables.

TOPIC 3. GRAPH OF AN EQUATION OF THE FORM $y = mx + b$

In this topic the student should discover the role of

(i) m, by graphing equations such as $y = x + 3$, $y = 2x + 3$, and $y = -4x + 3$, where $x \in R$, and

(ii) b, in a similar way.

He should be expected to re-express an equation such as $2x + 3y = 6$ in the form $y = -\frac{2}{3}x + 2$, but formal treatment of slope, y-intercept or families is not intended.

TOPIC 4. GRAPHS OF INEQUATIONS OF THE FORMS $y > mx + b, y < mx + b, y \geqq mx + b, y \leqq mx + b$

The student should now be ready to recognize that these inequations define relations (not functions) whose graph is a half-plane (open or closed) if $x \in R$. The defining inequations should be restricted to those in the form above.

TOPIC 5. GRAPHICAL SOLUTION OF A SYSTEM OF TWO FIRST DEGREE EQUATIONS IN TWO VARIABLES

From his earlier experiences in Topics 2 and 3, the student will readily accept the graphical determination of the solution set of the system. He should verify the solution by substitution. By informal discussion the student should realize that if the lines are parallel the solution set is empty. It is probably advisable not to discuss the case of two coincident lines. The exercises should include ones for which the co-ordinates of the ordered pair of the solution set are not integers and cannot be easily read from the graph; this will reinforce the need for verification, and create the need for a method of algebraic solution.

TOPIC 6. ALGEBRAIC SOLUTION OF A SYSTEM OF TWO FIRST DEGREE EQUATIONS IN TWO VARIABLES BY THE METHOD OF COMPARISON OF ORDINATES

This method follows most naturally from the fact that most of the equations have been given in the form $y = mx + b$ where $x \in R$. The student should recognize that for a given x the values of y determined from each equation are equal only at the point of intersection of the lines.

TOPIC 7. PROBLEM SOLVING (by the method of 6)

Generally, the problems selected should be practical ones, related to the branch of the students and not mechan-

ically difficult. The emphasis should be on the analysis leading to the development of the equations rather than memorization of procedures for type problems.

UNIT 4:
PURCHASING PERSONAL PROPERTY

The interest rate on credit purchases is a topic of universal importance. All students in the Four-Year Program should be able to decide whether instalment buying is worth the cost.

This unit is most meaningful when treated as a project. The student who collects a file of promissory notes, payment plan booklets, conditional sales agreements, finance company circulars and similar data will acquire first-hand knowledge of the credit method of purchasing personal property. The cost of credit should be discussed and the approximation to the true interest rate should be calculated wherever possible.

A comparison of the advantages of borrowing money from banks, credit unions and finance companies is important from a discussion point of view as well as for the mathematics involved. The use of specific terms such as deferred payments, revolving credit and effective interest rate is an excellent way to bring out the practical aspect of this topic.

UNIT 5:
RATIO AND PROPORTION

Topic 1. Two- and Three-Term Ratios

The notion of a two-term ratio $a:b$ has been introduced in Grades 7 and 8, where it is treated as equivalent to the rational number $\frac{a}{b}$, if $b \neq 0$. In this unit the notion of ratio is generalized to more than two terms, zero not excluded, e.g. inheritance, where one of the parties receives nothing.

Topic 2. Equal Ratios Leading to a Definition of Proportion

This extension in Topic 1 necessitates a new definition of equality of ratios, one that does not depend upon division. For example, if $a:b:c = d:e:f$, *then* $a = kd$, $b = ke$,

$c = kf$, for some real $k \neq 0$. This can be further generalized to equal ratios of 4 terms, or more, and the meaning of proportion follows. Students should discuss the need for the new more generalized definitions, and should develop that if $a:b:c = d:e:f$ then $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$ provided that $d, e, f \neq 0$, and interpret such statements as the circumference of a circle is *proportional* to its radius. Exercises should test the principles rather than manipulative skills, and emphasize proportions involving 2-term ratios. This would be a suitable time to discuss the use of the slide rule for computing in ratio and proportion problems.

Topic 3. Direct Variation as a Linear Function

This follows immediately from the meaning of proportion as discussed in Topic 2, and could be developed from examples which would relate to the students' experiences. The problems should be solved using both the proportion method, and the 'definition-equation' method. Partial variation might be considered at this time with a good class.

Topic 4. Inverse Variation

This topic follows naturally after Topic 3 by the same techniques.

Topic 5. Problems

It is intended that problems be blended in with the development of the above topics. Partial variation should be treated informally and can be represented by an equation of the form $y = mx + b$.

UNIT 7:
PAYROLL

The topic should consider all computational phases from the time employees are hired until they leave that employment.

A bulletin board display of all the forms used in payroll work would be most helpful. Students should be encouraged to bring the forms to be displayed. This display might include the following samples: application forms, personnel record cards, TD-1 income tax deduction declarations, time

cards, piece rate cards, incentive pay work tickets, incentive pay rate schedules, time clock illustrations, income tax deduction tables, social insurance booklets, social insurance handbooks, vacation pay stamp books, earning record cards, payroll summaries, currency analysis cards, currency analysis summaries, etc.

A planned visit to a local industrial plant for an explanation of their payroll procedures could be of value.

Accuracy in calculations and neatness are of prime importance in payroll work.

With a thorough understanding of the payroll procedures, the student should be well-equipped for payroll procedures which occur in machines courses.

UNIT 8:

SYSTEMS OF NUMERATION

As an introduction to this unit it might be worthwhile to discuss the origin of our present numbering system. Although our numerals are sometimes referred to as Arabic, this is a misnomer. They come to us, with many changes, from the Hindus. The Arabs were merely the intermediaries in this transition.

If man had had six fingers, the Hindus might have used six or twelve as the number base.

Some primitive tribes seemed to have used two as a base, and the use of this base has had important applications in recent years.

Today a binary notation has made possible electronic computers which use only the symbols 1 and 0, as the 'on' and 'off' in solid-state electronic devices.

In calculating machines based on the binary system it is necessary initially for the machine to convert decimal numbers to binary form, complete any necessary calculations, and finally to reverse the process.

Octal notation, using eight as the base, is convenient for interpreting binary numbers. By dividing the digits into groups of three, binary numbers may be converted to octal notations.

In the same way hexadecimal notation is convenient. By dividing the digits into groups of four, binary numbers may be converted to hexadecimal.

The use of Binary Coded Decimal Notation to express the alphabet and certain other useful characters will assist in the understanding of information storage in a computer's memory.

UNIT 10:

NUMERICAL APPLICATIONS OF GEOMETRY

Topic 1. Congruency, Parallelism, Angle Sum of a Triangle, and Pythagorean Relationships

Topic 2. Numerical Problems

Topics 1 and 2 should be integrated, and form a review of the inductive discoveries in the Grades 8 and 9 courses, with some extensions; for example congruency by A.A.S., exterior angle of a triangle, common Pythagorean triples (3,4,5), (5,12,13), (7,24,25), (8,15,17). The student should be encouraged once more to use a slide rule, square-root tables and a calculator with numerical problems.

Topic 3. Construction of Solids

This topic is suitable for a Mathematics Laboratory environment. Class time should be allotted for drawing nets such as and the construction of regular polyhedra, cones, pyramids, and prisms.

Suggested References

Cundy, H. M., and Pollett, A. P.,
Mathematical Models, Oxford

Johnson, Donovan A.,
Paper Folding for the Mathematics Class, N.C.T.M.

Suggested Materials

Paper, wood, pipe cleaners inserted in plastic straw, floor tiles joined with masking or adhesive tape, welding rods, sheet metal, tinker toys

(Projects to demonstrate Euler's Formula $V-E+F=2$ might be given.)

UNIT 11:
TRIGONOMETRY

General Suggestions

This topic should be taught in the late spring when groups of students can be outside using a surveyor's transit, gravity protractor, survey stakes, pickets, and 100-foot steel chain to collect data for use in the solution of right-angled triangles. Specific assignments should be made to determine inaccessible heights and distances. Computations should be done on a desk calculator or on a slide rule. Wherever possible, a Mathematics-Lab approach should be used where students have assigned projects to be handed in and graded.

Topic 1. Definition of the Primary Trigonometric Ratios $\sin\theta$, $\cos\theta$, $\tan\theta$, Where θ is Any Angle.

The general definitions should be given for any point $P(x,y)$ on the circle with centre the origin, radius r. Each definition will need to be interpreted in each quadrant with respect x, y, r, the sign of the ratio, and the change in value as θ increases. A 'trig-tracker' would assist the development of this lesson. It is not intended that negative values of θ be considered unless required by student discussion. This introduction could also be developed from a programmed sequence.

Topic 2. Relationships of These Ratios to Right-Angled Triangles

Topic 3. Tables of Values of Trigonometric Ratios of Acute Angles

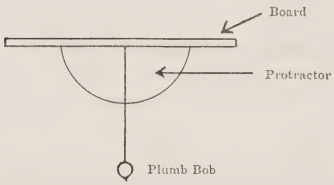
These two topics should be integrated. The definitions of Topic 1 can now be applied to right-angled triangles, so that the traditional definitions in terms of opposite, adjacent and hypotenuse may be given, independent of the cartesian coordinate system. Some drill might be given here on finding the third side of a right-angled triangle in order to find the three primary trigonometric ratios.

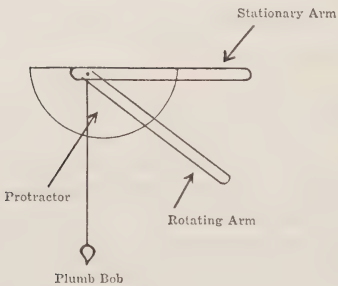
The tables of trigonometric ratios should now be introduced for angle $\theta°$ where $0° \leqq \theta° \leqq 90°$. The student might be first encouraged to develop his own tables for 5° intervals using a large circle, ruler, protractor and slide rule; he might then have a better understanding of the tables.

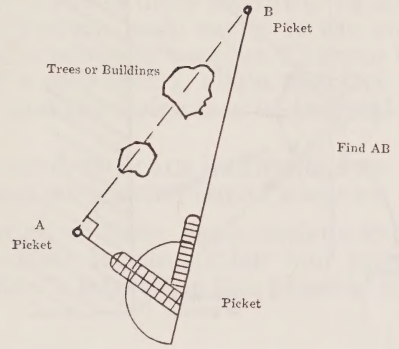
Exercises should now include ones requiring the student to find (i) the ratios, given the angle, (ii) the angle, given a ratio, (iii) extensions of (i), (ii).

TOPIC 4.

This topic could be discussed by a lab workshop approach using geometric models and problems related to 'shop work'. Assignments should be given to measure inaccessible heights, and angles, by using measuring instruments which could be designed and constructed by students, as illustrated by following diagrams:



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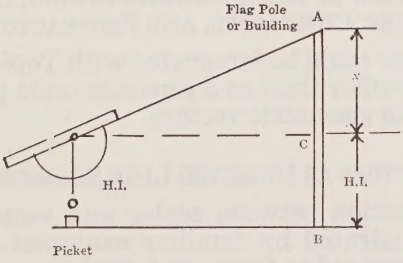
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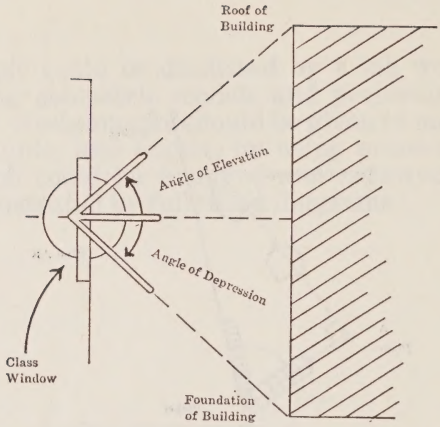

UNIT 12:

GEOMETRIC VECTORS (3 weeks)

Topic 1. Review of Basic Constructions, Using Ruler, Compasses, and Protractor

This review could be integrated with Topics 2, 3, as the need occurs, rather than as a separate topic preceding the introduction to geometric vectors.

Topic 2. Vectors as Directed Line Segments

The distinction between scalar and vector quantities should be illustrated by familiar examples such as age, volume as compared to force, and displacement, leading to the directed line segment representation of a vector.

Topic 3. Addition of Vectors by Scale Drawings

The addition of vectors could be developed with examples from physics. Consideration should be given to vectors that act in (i) the same direction, (ii) opposite direc-

tions, (iii) non-collinear directions. The treatment should be intuitive and lead to an understanding of the parallelogram and triangle laws of addition. Exercises should include an introduction to the addition of three or more vectors by a vector polygon.

Topic 4. Multiplication of a Vector by a Scalar

The addition of two or more vectors of equal magnitude and the same direction leads easily to the notion of the multiplication of a vector by a scalar. This can be extended to the notion of a zero vector, and a negative vector, and thus to the idea that the scalar may be any real number.

Topic 5. Extension to Scale Drawings for Navigation and Physics Problems

The application of these basic notions to illustrate a variety of problems in navigation and physics should awaken the student's interest in this phase of his studies.